C. Electron Spin

- Experimentally, electron carries an intrinsic spin magnetic moment (Stern-Gerlach exp't) $\vec{\mu}_s$, its z-component can take on only 2 values $\pm\mu_B$.
- Interpreted as electron carrying an intrinsic spin angular momentum $\vec{S}$
  [Intrinsic: an electron's property, not depending on its motion (like its charge $-e$)]
- $|\vec{S}|$ = magnitude $= \sqrt{\frac{3}{4}}\,\hbar$ (exp'tal fact)

$$= \sqrt{\frac{1}{2}\left(\frac{1}{2}+1\right)}\,\hbar = \sqrt{s(s+1)}\,\hbar$$

$$|\vec{S}| = \sqrt{s(s+1)}\,\hbar$$ ← general form of the magnitude of anything[+] called angular momentum in QM

- $s = 1/2$ for an electron [spin quantum number]
  "electron is a spin-half particle"

[+] If something $\vec{J}$ is claimed to be an angular momentum, then $J^2 = j(j+1)\hbar^2$ and

$$J_z = m_j\hbar \quad \text{with } m_j = j, j-1, \ldots, -j+1, -j$$

$$S_z = \begin{cases} \frac{1}{2}\hbar & (m_s = +1/2) \quad \text{"spin up"} \\ -\frac{1}{2}\hbar & (m_s = -1/2) \quad \text{"spin down"} \end{cases}$$

[What is "z"-direction?] (Nothing special!)
[Any component has eigenvalues $+\frac{\hbar}{2}, -\frac{\hbar}{2}$]

One possible representation is:

z-component of $\vec{S}$

$$S_z = \frac{\hbar}{2}\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}}_{\sigma_z}$$

Eigenvalues $\nearrow \frac{\hbar}{2}$, $\searrow -\frac{\hbar}{2}$

S_z: Operator of z-component of spin

σ_z: z Pauli matrix

(Similar expressions for S_x, S_y)

- For eigenvalue $+\frac{\hbar}{2}$, eigenvector is $\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \alpha$ (or α_z) = spin up state
- For eigenvalue $-\frac{\hbar}{2}$, eigenvector is $\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \beta$ (or β_z) = spin down state

We will use the notations α and β for spin-up state and spin-down state, respectively later in the course.

- With $s = 1/2$, the z-component of the spin angular momentum S_z can take on

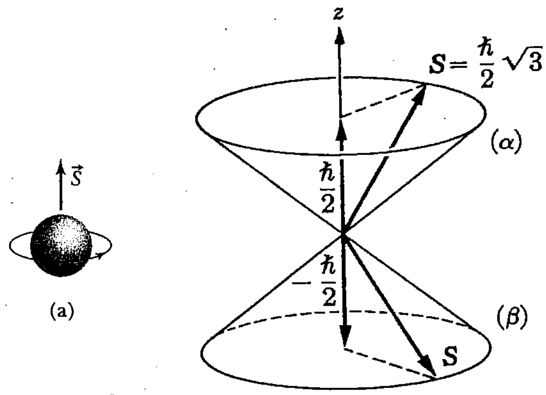
$$S_z = \begin{cases} \frac{1}{2}\hbar & (m_s = +1/2) \\ -\frac{1}{2}\hbar & (m_s = -1/2) \end{cases}$$

- Solutions to H-atom + Spin — essential to the understanding of the periodic table and much chemistry
- Scientists involved: Pauli, Stern, Gerlach, Goudsmit, Uhlenbeck
- Within Schrödinger Equation, spin is NOT there! We need to insert spin into the solutions to TISE. It is an add-on feature. Thus, the existence of spin is an embarrassment of Schrödinger quantum theory.
- Dirac's theory of an electron gives spin within the theory (and requires anti-electron's presence). The point is that spin originates from relativistic effect.

- Spin is a quantum and relativistic entity. No way to consider it classically. Yet it is useful to have a "picture" in mind

Vector Model

(a) A purely classical schematic of the intrinsic spin angular momentum, $\vec{S}$, of a spinning electron. (b) The quantization of $\vec{S}$, which can have only two positions in space relative to z (direction of external magnetic field). The z component of $\vec{S}$ is $S_z = \pm\hbar/2$.



(From Thorton and Rex) The two-spin state of an electron, α = "spin up," and β = "spin down."

Just a classical picture, don't take it too seriously!

- Later found all particles carry the spin quantum number s

Think like a physicist

- Electronic devices changed the world!
- Using (−e) property of electrons
- Spintronic devices? ∘ spin-diode? ∘ spin transistor?

  use spin property of electrons

  (e.g. computer hard-disks)

[See Awschalom et al., Scientific American (June 2002) for an introduction]

Remarks

- 1D, 2D, 3D problems in Sec. B and electron spin in Sec. C are covered in QMI. We will use the results in our course.

- For a review (or if you want to learn them again), see
  - Introduction to Quantum Mechanics (Griffiths)
  - Quantum Mechanics (Rae)
  - Quantum Physics (Gasiorowicz)

  [all reserved in library]